

9. A solid of revolution is formed by rotating about the x -axis, the area between the x -axis, the lines $x = 0$ and $x = 1$ and a curve through the points with the following co-ordinates :

$x :$	0.00	0.25	0.50	0.75	1.00
$y :$	1.0000	0.9898	0.9589	0.9089	0.8415

Estimate the volume of the solid formed using Simpson's rule.

10. Solve $y_{n+2} - 4y_{n+1} + 3y_n = 5^n$.

(5 × 5 = 25 marks)

Part C

Answer any one full question from each module.
Each full question carries 12 marks.

Module 1

11. (a) If $r = \sqrt{x^2 + y^2 + z^2}$, show that $\nabla^2(r^n) = n(n+1)r^{n-2}$ and hence deduce that $\nabla^2\left(\frac{1}{r}\right) = 0$, except at $r = 0$. (6 marks)

- (b) Show that $\text{curl}(\phi \bar{A}) = \text{grad } \phi \times \bar{A} + \phi \text{curl } \bar{A}$. (6 marks)

Or

12. (a) Find the directional derivative of $\nabla \cdot (\nabla \phi)$ at the point $(1, -2, 1)$ in the direction of the normal to the surface $xy^2z = 3x + z^2$, where $\phi = 2x^3y^2z^4$. (6 marks)

- (b) Show that $\text{curl}(\bar{u} \times \bar{v}) = (\bar{v} \cdot \nabla)\bar{u} - (\bar{u} \cdot \nabla)\bar{v} + \bar{u} \text{div } \bar{v} - \bar{v} \text{div } \bar{u}$. (6 marks)

Module 2

13. Verify divergence theorem for $\bar{F} = (2xy + z)\hat{i} + y^2\hat{j} - (x + 3y)\hat{k}$ when the surface S is that of the region bounded by the plane $2x + 2y + z = 6$ in the first octant. (6 marks)

Or

14. Use Stoke's theorem to calculate $\oint_C (ydx + zdy + xdz)$, where C is the curve of intersection of $x + y = 2$ and $x^2 + y^2 + z^2 - 2x - 2y = 0$. (6 marks)

Module 3

15. Using Newton's forward interpolation formula, estimate the number of students who scored marks between 40 and 45 :

Marks	:	30—40	40—50	50—60	60—70	70—80
No. of students	:	31	42	51	35	31

Or