9. Solve $y_{x+2} - 4yx = 9x^2$.

10. Prove that $Z\left\{\frac{1}{n}\right\} = z \log \frac{z}{z-1}$.

(5 × 5 = 25 marks)

Part C

Answer any **one** *full question from each module.*
Each full question carries 12 *marks.*

Module I

11. (a) Find the directional derivative of $\phi(x, y, z) = 4xz^3 - 3x^2yz^2$ at (2, –1, 2) along the z-axis.

(5 marks)

(b) Prove that $\operatorname{div}\{\bar{f} \times \bar{g}\} = \bar{g} \cdot (\operatorname{curl} \bar{f}) - \bar{f} \cdot (\operatorname{curl} \bar{g})$. (7 marks)

Or

12. (a) Prove that $\bar{f} = (2x + yz)\bar{i} + (4y + zx)\bar{j} - (6z - xy)\bar{k}$ is both solenoidal and irrotational. Also find the scalar potential of $\bar{f}$.

(7 marks)

(b) Prove that $\nabla^2\left\{\nabla \cdot \left(\frac{\bar{r}}{r^2}\right)\right\} = 2r^{-4}$. (5 marks)

Module II

13. Verify Stoke's theorem for $\bar{F} = y\bar{i} + z\bar{j} + x\bar{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C its boundary.

(12 marks)

Or

14. Verify divergence theorem for $\bar{F} = 4xz\bar{i} - y^2\bar{j} + yz\bar{k}$ and S is the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$.

(12 marks)

Module III

15. Find y_{32} given y_{20} = 14.035, y_{25} = 13.674, y_{30} = 13.257, y_{35} = 12.734, y_{40} = 12.089 and y_{45} = 11.309.

(12 marks)

Or