G 1833 (Pages : 4) Reg. No..............................

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B.TECH. DEGREE EXAMINATION, MAY 2012

Fourth Semester

EN 010 401—ENGINEERING MATHEMATICS—III

(Regular—2010 Admissions)

[Common to all Branches]

Time : Three Hours Maximum : 100 Marks

Part A

Answer **all** *questions.*
Each question carries 3 marks.

1. Expand $\pi x - x^2$ in a half range sine series in the interval $(0, \pi)$ upto the first three terms.

2. Find the Fourier Transform of $f(x) = \begin{cases} 1 \text{ for } |x| < 1 \\ 0 \text{ for } |x| > 1. \end{cases}$

3. Form the partial differential equation by eliminating the arbitrary functions from

   $f(x + y + z, x^2 + y^2 + z^2) = 0.$

4. During war, one ship out of nine was sunk on an average in a certain voyage. What was the probability that exactly 3 out of a convoy of 6 ships would arrive safely ?

5. A random sample of 900 members has a mean 3.4 cm. Check if it can be reasonably regarded as a sample from a large population of mean 3.2 cm. and SD = 2.3 cm.

(5 × 3 = 15 marks)

Part B

Answer **all** *questions.*
Each question carries 5 marks.

6. Obtain Fourier series for the function

$$f(x) = \pi x, \qquad 0 \le x \le 1$$
$$= \pi(2 - x) \quad 1 \le x \le 2$$

7. Find the Fourier cosine transform of $f(x) = \dfrac{1}{1 + x^2}$ and hence derive Fourier sine Transform of $\phi(x) = \dfrac{x}{1 + x^2}$.

Turn over

8. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, given that $\frac{\partial z}{\partial y} = -2\sin y$, when $x = 0$ and $z = 0$, when y is an odd multiple of $\frac{\pi}{2}$.

9. Assume that the probability of an individual coal-miner being killed in a mine accident during an year is $\frac{1}{2400}$. Use Poisson's distribution to calculate the probability that in a mine employing 200 miners, there will be at least one fatal accident in a year.

10. A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased.

(5 × 5 = 25 marks)

Part C

*Answer any **one** full question from each module.*
Each full question carries 12 marks.

Module 1

11. If $f(x) = x, \quad 0 < x < \pi/2$
$= \pi - x, \quad \pi/2 < x < \pi$, show that

(a) $f(x) = \frac{4}{\pi}\left[\sin x - \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} - \ldots\ldots\right]$. (5 marks)

(b) $f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left[\frac{\cos 2x}{1^2} + \frac{\cos 6x}{3^2} + \frac{\cos 10x}{5^2} + \ldots\ldots\right]$. (7 marks)

Or

12. Obtain the first three coefficients in the Fourier Cosine series for y from the following data :

x :	0	1	2	3	4	5
y :	4	8	15	7	6	2

(12 marks)

Module 2

13. (a) Using Fourier integral representation, show that $\int_0^\infty \frac{\cos \omega x}{1+\omega^2}\, d\omega = \frac{\pi}{2} e^{-x} \, (x \geq 0)$. (6 marks)

(b) Solve for F (x) the integral equation $\int_0^\infty F(x)\, \sin tx \; dx = \begin{cases} 1, & 0 \leq t < 1 \\ 2, & 1 \leq t < 2 \\ 0, & t \geq 2. \end{cases}$ (6 marks)

14. (a) Using Parseval's identity, prove that $\int_0^{\infty} \frac{dt}{(a^2+t^2)(b^2+t^2)} = \frac{\pi}{2\,ab\,(a+b)}$. (5 marks)

(b) Solve the integral equation $\int_0^{\infty} F(x)\cos px = dx \begin{cases} 1-p, & 0 \le p \le 1 \\ 0, & p > 1 \end{cases}$ and hence deduce that

$$\int_0^{\infty} \frac{\sin^2 t}{t^2}\, dt = \frac{\pi}{2}.$$

(7 marks)

Module 3

15. Solve $2zx - px^2 - 2\,pxy + pq = 0$. (12 marks)

Or

16. Solve :

(a) $\left(D^2 - 2\,DD' + D'^2\right)z = e^{(2x+3y)}$. (6 marks)

(b) $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 12\,xy$. (6 marks)

Module 4

17. A random variable X has the following probability distribution values of X :

x :	0	1	2	3	4	5	6	7	8	9
$p(x)$:	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$	$19a$

(a) Determine the value of a (3 marks)

(b) Find $P(X<3), P(X \ge 3), P(2 \le X < 5)$. (6 marks)

(c) What is the smallest value for which $P(X \le x) > 0.5$? (3 marks)

Or

18. A sample of 100 button cells tested to find the length of life, produced the following results : $\bar{x} = 12$ hours, σ = 3 hours. Assuming the data to be normally distributed, what percentage of button cells are expected to have life

(a) more than 15 hours ; (4 marks)

(b) less than 6 hours ; and (4 marks)

(c) between 10 and 14 hours ? (4 marks)

Module 5

19. Two independent sample sizes of 7 and 6 has the following values :

Sample A	:	28	30	32	33	31	29	34
Sample B	:	29	30	30	24	27	28	—

Examine whether the samples have been drawn from normal populations having the same variance.

(12 marks)

Or

20. Records taken of the number of male and female births in 800 families having four children are as follows :

No. of male births	:	0	1	2	3	4
No. of female births	:	4	3	2	1	0
No. of families	:	32	178	290	236	94

Test whether the data are consistent with the hypothesis that the binomial law holds and the chance of male birth is equal to that of the female birth, namely, $p = q = \frac{1}{2}$.

(12 marks)

[5 × 12 = 60 marks]